# 1 A Formal Description of Backbone

In this section we formally describe how adding component definitions, defined using resemblance and replacement, adjusts the existing compositional structure of an architecture. As dependencies between strata govern the order of application and interplay between replacement and resemblance, we begin by elucidating the stratum concept.

The ability to evolve an existing architecture in a decentralized manner, using strata to group these component definitions, leads logically to the desire to merge independently developed strata back into a unified architecture. We describe the merging rules, showing that any resulting structural errors can be corrected by adding further component definitions.

## 1.1 Strata and Strata Dependencies

A stratum is a hierarchical module that owns and groups elements such as components and interfaces. Each stratum explicitly indicates its dependence on other strata. An element can only refer to other elements in its owning stratum, or those visible via transitive strata dependencies.

Definition 1 (Stratum): A stratum s is represented as the structure

where *p* indicates a single possible parent stratum giving rise to a hierarchy, *D* is the set of other strata that *s* depends on, and *E* represents the set of elements owned by *s*.

Definition 2 (Strata visibility): We define to be the reflexive-transitive closure of the dependencies . This represents all strata (and hence owned elements) visible to elements owned by . We further define to be the transitive closure of the dependencies, equivalent to .

Definition 3 (Strata non-circularity): Strata dependencies must form a graph.

Definition 4 (Strata independence): Two strata share a common base, but are independent, if neither have visibility of the other via their dependencies but each has visibility of common strata. This allows us to model two strata developed in isolation in a possibly decentralized manner, which nevertheless build on and possibly evolve elements in a common set of base strata.

## 1.2 Elements

Definition 5 (Element): An element is represented as the structure:

where is the owning stratum, is the optional element that this definition replaces and represents the set of elements being resembled. The element being replaced cannot be from the same stratum This accords with the use of stratum as a unit of ownership: if the owner of a stratum wishes to alter an element within that stratum, then they can do this directly via destructive editing.

Definition 6 (Single replacement): A stratum cannot contain two elements which both replace the same element. Without this restriction, we could potentially have multiple replacements inside a single stratum, which could not be otherwise ordered.

An element’s expanded structure is determined by applying deltas ( to the structure inherited from the elements it resembles, resulting in a set of constituents. For instance, the constituents of a component are port, part, connector and attribute. represents the set of constituents added by this definition, the set of inherited constituents which are to be deleted, and is an injection indicating a subset of inherited constituents to be replaced by new constituents.

## 1.3 Interplay Between Replacement and Resemblance

As previously indicated, a stratum can contain elements definitions which replace elements in other strata. This can affect existing resemblance relationships, as an element which was previously being resembled may be replaced with another definition. As such, we need to re-determine the resemblance graph for the entire system anew from the perspective of each stratum.

Definition 7 (Simple dependencies): We define to remove any redundancy (strata otherwise reachable transitively) from the set of dependencies.

Definition 8 (Expanded resemblance): We then define to be the elements resembled by , from the perspective of stratum , after taking any replacements from the strata set into account.

This function can be used to determine an expanded resemblance graph for an element, from a given stratum. Note that this may result in a multi-headed graph, if a stratum depends on independent strata which both replace the same element.

## 1.4 Applying Deltas to Form the Expanded Set of Constituents

Using the expanded resemblance graph, we can now combine the deltas for each element to form its expanded definition from each stratum perspective.

(recursive...)

## 1.5 Merging Independent Strata

## 1.6 Correcting Structural Merge Errors